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LOSS OF STABILITY OF THIN ELASTIC SHELLS UNDER THE EFFECT OF IMPULSIVE LOADS

(O POTERE USTOICHIVOSTI TONKIKH UPRUGIKH OBOLOCHEK POD DEISTVIEM IMPULSIVNOI NAGRUZKI)

by

V. V. Bolotin, G.A. Boichenko, B. P. Makharov,
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Translation No. 73

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We propose to discuss the problem of the stability of a cylindrical
panel resting on a rectangular boundary, under the effect of a normal im-
pulse which is characterized by a rapid increase in load to a certain magni-
tude and subsequent decrease according to the exponential law. Our main ob-
jective is to construct a range of parameters which characterizes this im-
pulse and in the presence of which snap-through buckling of the shell does
not occur (i.e., stability "in the large"). The effect of the initial com-
pression and damping decrements on the boundary of the stability region, as
well as on the maximum deflections attainable under the effect of the im-
pulse are investigated.[1]

Partial differential equations of the nonlinear shell theory are, by
applying the Papkovich-Galerkin method, reduced to nonlinear ordinary dif-
ferential equations and solved on continuously-acting analog computers (in
this work the MN-7 computer was used).

1. Basic Equations. We shall use the basic hypotheses of the non-
linear theory of thin elastic shells [1, 2] assuming that elongations and
shears are small in comparison with unity and that the deflections of the
shell are comparable with its thickness, though small in comparison with its
other dimensions. We shall also assume that the Kirchhoff-Love hypothesis
remains valid in the case of finite deflections of the shell as well.

Under the assumptions made, the deflection equations of the shell
with principal curvatures k_1 and k_2 have the following form:

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[1] One of the authors presented this paper at the conference (3) on elastic
oscillations at the Institute of Mechanical Engineering of the Academy of
Sciences of the Latvian S.S.R. (Riga, June 1958).

$$D\nabla^2\nabla^2 w = L(w + w_0, \Phi) + k_1 \frac{\partial^2 \Phi}{\partial x^2} + k_2 \frac{\partial^2 \Phi}{\partial y^2} + q$$

$$\frac{1}{Eh}\nabla^2\nabla^2\Phi = -\frac{1}{2}L(w, 2w_0 + w) - k_2 \frac{\partial^2 w}{\partial x^2} - k_1 \frac{\partial^2 w}{\partial y^2} \qquad (1.1)$$

Here $w_0(x,y)$ is the initial deflection, $w(x,y,t)$ is the complementary deflection, x and y are the coordinates computed along the lines of curvature in the middle surface, D is the flexural rigidity, E is the modulus of elasticity, h is the thickness of the shell, $\Phi(x,y,t)$ is a stress function of the middle surface stress-resultants related to a unit length.

$$N_x = \frac{\partial^2 \Phi}{\partial y^2}, \; N_y = \frac{\partial^2 \Phi}{\partial x^2}, \; N_{xy} = -\frac{\partial^2 \Phi}{\partial x\,\partial y} \qquad (1.2)$$

The symbol $L(A,B)$ denotes a bilinear operator

$$L(A,B) = \frac{\partial^2 A}{\partial x^2}\frac{\partial^2 B}{\partial y^2} + \frac{\partial^2 A}{\partial y^2}\frac{\partial^2 B}{\partial x^2} - 2\frac{\partial^2 A}{\partial x\,\partial y}\frac{\partial^2 B}{\partial x\,\partial y}$$

If the shell oscillates, then the normal load component q will be computed using the formula

$$q(x,y,t) = -\rho h \frac{\partial^2 w}{\partial t^2} - 2\rho h \epsilon \frac{\partial w}{\partial t} + q_0(x,y,t) \qquad (1.3)$$

where ρ is the density of material of the shell, ϵ is the damping coefficient, and q_0 is the external load.

We note that along with normal inertia forces, the tangential inertia forces should be considered in the equation as well. However, in the case where loading is not too fast, i.e., if the time for the external forces to vary by an appreciable magnitude is sufficiently long in comparison to the longest period of tangential natural oscillations though commensurable with the longest period of lateral oscillations, then tangential forces of inertia can be disregarded. An analogous assumption is usually introduced in the investigation of natural and forced oscillations of plates and shallow shells [4].

To solve the system of equations (1.1) for a case of normal load defined by formula (1.3) we shall use the Papkovich-Galerkin method.

We shall represent the initial and complementary deflections in the form of series of functions $\psi_j(x,y)$ satisfying the boundary conditions

$$w_0(x,y) = \sum_{j=1}^{\infty} \zeta_j^{(o)} \psi_j(x,y)$$

$$w(x,y,t) = \sum_{j=1}^{\infty} \zeta_j(t) \psi_j(x,y) \tag{1.4}$$

Here, $\zeta(t)$ are the unknown time functions.

Functions $\psi_j(x,y)$ are selected in such a way as to satisfactorily describe the expected mode shape of the oscillations of the shell; in the following we shall assume that the mode shape of the small natural oscillations of the shell are such functions. At first, we shall substitute the series (1.4) into the second equation (1.1) and solve it; we shall then satisfy boundary conditions of the stress function $\Phi(x,y,t)$ and substitute it into the first equation. Finally, we will apply the Galerkin variational method, and the problem will be reduced to a system of ordinary differential equations

$$\frac{d^2 \zeta_j}{dt^2} + 2\epsilon_j \frac{d\zeta_j}{dt} + \omega_j^2 \zeta_j + f_j(\zeta_1,\zeta_2,\ldots,\zeta_m,t) =$$

$$q_j(\zeta_1^{(o)},\zeta_2^{(o)},\ldots,\zeta_m t) \qquad j = 1,2,\ldots \tag{1.5}$$

Here ϵ_j denotes damping coefficients which, in contrast to formula (1.3), are assumed to be distinct, and ω_j denotes the frequencies of small natural oscillations. Nonlinear terms accounting for external load are included in functions f_j and q_j.

In the case of a shell with edges freely supported on a contour with sides a and b (Figure 1), the solution can be written in the form

$$w_0(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \zeta_{mn}^{(o)} \sin \frac{m\pi x}{a} \sin \frac{n\pi v}{b}$$

$$w(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \zeta_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

The number of the retained terms of the series determines the order of the differential system (1.5) and thus is controlled by the capacity of the analog computer. The MN-7 computer can solve nonlinear equations of the second order; accordingly we write the solution in the form

$$w_0(x,y) = \zeta_0 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$$

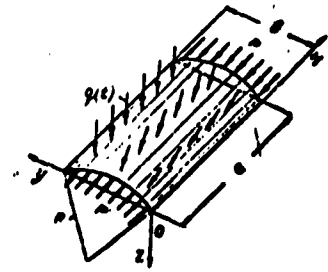$$w(x,y,t) = \zeta(t) \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \qquad (1.6)$$

Figure 1

Substituting expression (1.6) into the second of the equations (1.1) we obtain equations of the following type:

$$\nabla^2\nabla^2\Phi = F(x,y,t)$$

(here, time t is considered as a parameter).

Let us consider the case of a cylindrical shell $(k_1 = 0, k_2 = 1/R)$ with the following boundary conditions: edges $x = 0$ and $x = a$ are under compressive stresses $N_x = -p$, while edges $y = 0$ and $y = b$ are restrained from displacements. The tangential stresses on the edges will be considered equal to zero. If one takes for Φ the expression

$$\Phi = \frac{Eh}{32} (\zeta + 2\zeta_0)\zeta\left(\frac{a^2}{b^2} \cos \frac{2\pi x}{a} + \frac{b^2}{a^2} \cos \frac{2\pi y}{b}\right) +$$

$$\frac{Eh\zeta}{\pi^2 a R}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^{-2} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} - \frac{p_x y^2}{2} - \frac{p_y x^2}{2} \qquad (1.7)$$

then these boundary conditions will be satisfied "in the mean."

Here, p_x and p_y are constants which can be determined from the conditions

$$\frac{1}{b}\int_0^b \frac{\partial^2\Phi}{\partial x^2} \, dy = - p$$

$$\int_o^a \int_o^b \left[\frac{1}{Eh} \left(\frac{\partial^2 \Phi}{\partial x^2} - \mu \frac{\partial^2 \Phi}{\partial y^2} \right) + \frac{w}{R} - \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 + \frac{\partial w}{\partial y} \frac{\partial w_o}{\partial y} \right] dx\ dy = 0 \tag{1.8}$$

The meaning of the first condition is explicit. The second condition requires that the displacement of edges $y = 0$ and $y = b$ in the direction of y axis is, "on the average," equal to zero.

Substituting expressions (1.6) and (1.7) into conditions (1.8) we obtain constants p_x and p_y, after which the function (1.7) can be completely determined.

We then substitute the expression for this function into the first of equations (1.1). In order to reduce this equation to an ordinary differential equation we substitute into it the expressions (1.6) and we apply the Galerkin variational method. After the usual computation, which is not shown here, we obtain the following equation:

$$\frac{d^2 \zeta}{dt^2} + 2\epsilon \frac{d\zeta}{dt} + \omega^2 (1 - \frac{p}{p_o})\zeta + f(\zeta, \zeta_o) =$$

$$\frac{\omega^2 p}{p_o} \zeta_o - \nu\omega^2 \frac{p}{p_o} + 16 \frac{q_o \Phi(t)}{\pi^2 \rho h} \tag{1.9}$$

Here, ω is the frequency of small natural oscillations of an unloaded shell, $\zeta(t)$ is a function characterizing the response, $f(\zeta, \zeta_o)$ is a nonlinear function which is a part of the representation of initial and subsequent deflections, p_o is a magnitude having linear stress dimension, and q_o is a certain constant characterizing the magnitude of normal load (for example, its value for $t = 0$).

$$\omega = \frac{\pi^2}{a^2} \sqrt{\frac{D}{\rho h}} F(n,k), \quad p_o = \frac{\pi^2 D}{a^2 (1 + \mu n^2)} F(n,k) \tag{1.10}$$

$$\nu = \frac{16\mu k}{\pi^4 (1 + \mu n^2)}, \quad n = \frac{a}{b}, \quad k = \frac{a^2}{Rh} \tag{1.11}$$

$$F(n,k) = (1 + n^2) + \frac{12k^2 (1 - \mu^2)}{\pi^4 (1 + n^2)^2} \left[\frac{64}{\pi^4} + \frac{1 - 64\pi^4 (1 - \mu n^2)}{(1 + n^2)^2} \right] \tag{1.12}$$

$$f(\zeta, \zeta_o) = \frac{\omega^2 \zeta^3 \alpha}{h^2} + \frac{\omega^2 \zeta^2 \beta(\zeta_o)}{h} + \omega^2 \zeta \gamma(\zeta_o) \tag{1.13}$$

$$\alpha = \frac{3}{4} \frac{1 - \mu^2}{F(n,k)} (1 + 3n^4) \qquad (1.14)$$

$$\beta(\zeta_0) = \frac{1 - \mu^2}{F(n,k)} \left\{ \frac{9}{4} (1 + 3n^4)\zeta_0 + \frac{16kn^2}{\pi^4} \left[\frac{3(1 - \mu n^2) - 8}{(1 + n^2)^2} - 5 \right] \right\}$$

$$\gamma(\zeta_0) = \frac{1 - \mu^2}{F(n,k)} \left\{ \frac{3}{2} (1 + 3n^4)\zeta_0^2 + \frac{16kn^2}{\pi^4} \left[\frac{3(1 - \mu n^2) - 8}{(1 - n^2)^2} - 7 \right]\zeta_0 \right\}$$

For what follows, we shall introduce the following dimensionless variables:

$$\frac{\zeta}{h} = \bar{\zeta}, \quad \frac{\zeta_0}{h} = \bar{\zeta}_0, \quad \omega t = \tau$$

$$\frac{2\pi\epsilon}{\omega} = \delta, \quad \frac{16q_0}{\pi^2\omega^2\rho h^2} = \bar{\zeta}' \qquad (1.15)$$

The meaning of the first three variables is obvious. The next to the last variable is a decrement of the small natural oscillations and the last variable represents a deflection of the center of the shell relative to its thickness under the effect of a load q_0 applied statically. The equation (1.9) takes the form

$$\frac{d^2\zeta}{d\tau^2} + \frac{\delta}{\pi} \frac{d\zeta}{d\tau} + \left(1 - \frac{-p}{p_0} \right)\zeta + \alpha\zeta^3 + \beta\zeta^2 + \gamma\zeta =$$

$$\frac{p}{p_0}\zeta_0 - \frac{\nu p}{p_0} + \zeta'\varphi(\tau) \qquad (1.16)$$

Here, and in that which follows, the bar over ζ will be omitted.

2. <u>Results of the Solution Obtained by Means of an Electronic Analog Computer.</u> The equation (1.16) will be solved for the case where the impulse has an exponential form, i.e., for the case, where

$$\varphi(\tau) = e^{-c\tau} \quad \text{for} \quad \tau \geq 0(\omega t = \tau) \qquad (2.1)$$

Here $\dot{c}$ is a constant characterizing the rate of decrease of load intensity. For example, if $cn = 0.5$, this means that intensity of the load decreased by a factor e within one period of the free linear oscillations. According to formulae (2.1), the load q increases instantly from 0 to q_0 at the instant of time $t = 0$. In addition, it is necessary to take into account both normal and tangential inertia forces. A contradiction will be eliminated if one considers that the expression describes an idealized picture in comparison with actual conditions when pressure increases rapidly, yet with a finite velocity (Figure 2).

We shall not discuss here the effect of tangential inertia forces and the effects of elastic wave propagation occurring with large deflections of the shell.

If an elastic shell is subjected to impulsive loads, the following problems originate:

a) to find the conditions under which buckling of the shell would not occur,

Figure 2

b) if buckling is impossible, then to find the greatest deflections and stresses originating with an impact,

c) if buckling occurs all the same, then to find the greatest deflections and stresses which occur after buckling.

Computations were made for a cylindrical panel resting on a square contour $(n = 1)$, with $\mu = 0.3$ and $k = 12$. In this case, formulae (1.14) yield

$$\alpha = 0.162, \quad \beta = 0.485\zeta_0 - 0.690, \quad \gamma = 0.324\zeta_0^2 - 0.905\zeta_0$$

When impacted, the shell's behavior depends essentially on the magnitude of the initial load p. In order to investigate this problem we shall first discuss equation (1.16) for the case of a static load. For an ideal shell $(\zeta_0 = 0)$, the equation takes the form

$$\left(1 - \frac{p}{p_0}\right)\zeta + \alpha\zeta^3 + \beta\zeta^2 = \zeta' - \frac{p}{p_0}\nu \tag{2.2}$$

Dependence of the dimensionless deflection ζ on ζ' for various ratios p/p_0 are shown in Figure 3. It follows from this graph that for sufficiently small values of p (approximately for $p < 0.3p_0$) and for $\zeta' = 0$, there exists one elastic equilibrium state of the shell. For
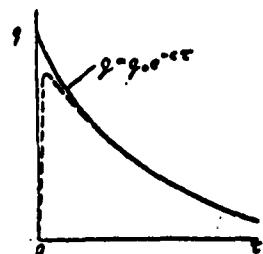
$p > 0.3p_0$ and $\zeta' = 0$ there are three equilibrium states; one of them corresponds to the negative root of equation (2.2), i.e., to a deflection of the shell with an increase of its curvature; the other two correspond to the positive roots of this equation.

The intermediate root corresponds obviously to the unstable form of equilibrium. We note that the upper critical pressure does not, in the usual sense of the word, exist in this case, since as a result of Poisson's effect the shell deflections resulting from curvature increase are present even with small values of p.

For $\zeta' = 0$, the roots of equation (2.2) become, with $\tau \to 0$, the singular points of equation (1.16). Thus, for a square panel and with $p <$ $0.3p_0$ there is one singular point (stable focus), while with $p > 0.3p_0$ there are three singular points (two stable foci and a saddle). With $p <$ $0.3p_0$, and regardless of magnitude of ζ', there is only one type of motion possible, namely, oscillations with a gradually decreasing amplitude around a unique stable focus, i.e., oscillations without buckling. With $p > 0.3p_0$, the following three types of motion are possible: 1) for small ζ' -- oscillations without buckling around a focus nearest to the origin of coordinates; 2) for large ζ' -- buckling with subsequent oscillations around the second focus; 3) for very large ζ' -- buckling with subsequent oscillations along phase trajectories embracing all three singular points. In the last case, as $t \to \infty$, generally speaking, convergence to any of the two stable foci is possible.
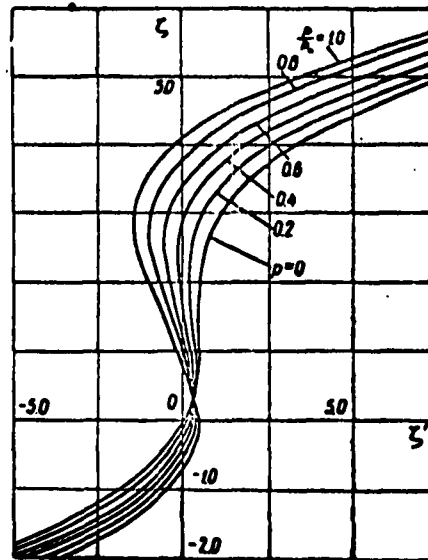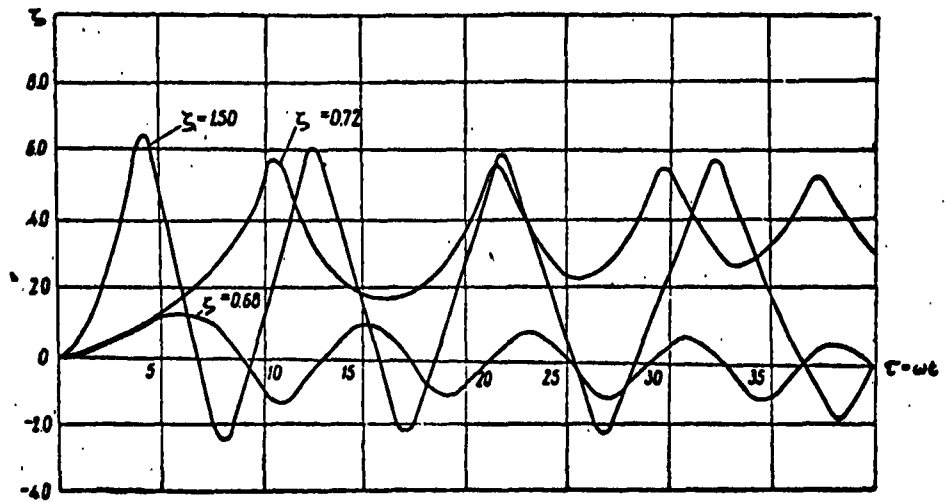

Figure 3

Figure 4

Figure 4 shows behavior of the shell with $\zeta' = 0$, $p = 0.5p_o$, $\delta = 0.05\pi$, and $c\pi = 2.0$.

If $\zeta' = 0.68$, buckling does not occur; with $\zeta' = 0.72$, there is buckling with oscillations around the second focus; with $\zeta' = 1.50$, oscillations occur with a change of deflection sign.

The phase plane diagram of this case is shown in Figure 5 (after several oscillation cycles, damping with a higher than critical magnitude was added; this was done in order to accelerate the approach of the system to a stable singular point).

It is of interest to construct boundaries of a c, ζ' domain of parameters with such a characteristic that when the shell is under the effect of an impulse having parameters lying in this domain, buckling of the shell will not occur. Figure 6 shows the boundaries of such stability regions for an ideal shell for various magnitudes of a longitudinal compressive stress. It is clear that stability regions are located on the side of smaller values of ζ'.
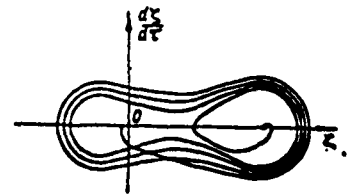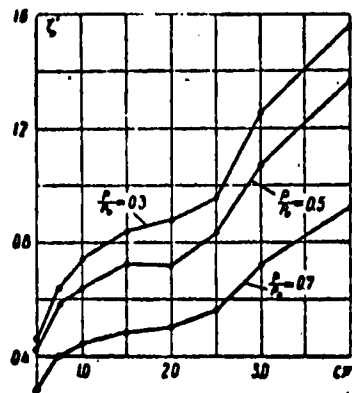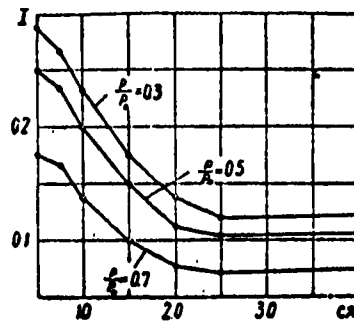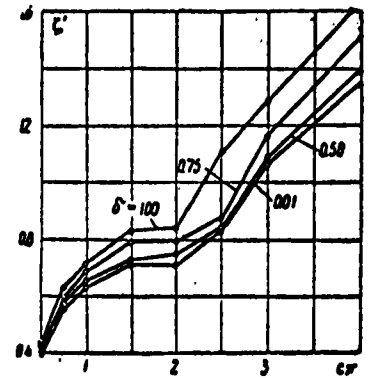

Figure 5

Figure 6

Figure 7

Figure 8

It would be of interest to learn the dependence of the critical impulse on other parameters of the problem. It is easy to see that a dimensionless impulse I can be determined from the condition

$$I = \zeta' \int_0^\infty e^{-c\tau} d\tau = \frac{\zeta'}{c}$$

The graph shown in Figure 7 was plotted according to this formula.

Figure 8 shows the effect of damping decrement δ on the boundaries of the stability regions. As was expected, this dependence turns out to be weak, particularly when damping is small. Thus, when damping increases from δ = 0.01 to δ = 0.50, the critical value of parameter ζ' increases no more than 5%.

Figure 9 and Figure 10 show the dependence of the greatest deflection ζ_{max} of the shell on the intensity of a normal load ζ for values $p = 0.2p_0$ and $p = 0.5p_0$. In the second case, during transition of ζ' through the critical value, the maximum deflections increase sharply; however, with sufficiently strong impulses they differ, in both cases, only slightly from each other. On the other hand, the difference in behavior of the shell remains significant with τ → ∞. In this case, for $p < 0.3p_0$, and after damping of oscillations, the shell returns to the initial state of equilibrium; however, for $p = 0.3p_0$, the most probable form of equilibrium of the shell after damping of oscillations appears to be the buckled state.
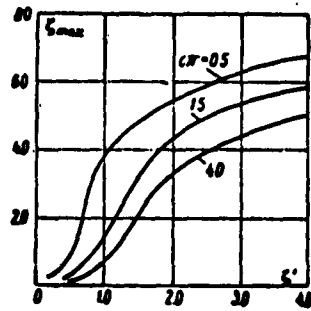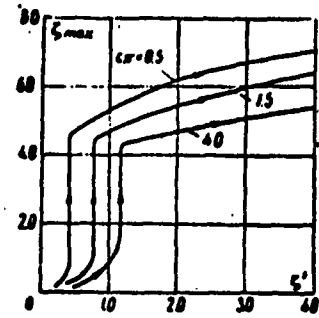
Figure 9

Figure 10

From the above a conclusion can be drawn that an elastic shell can withstand, without buckling, impulsive loads exceeding several times the critical static load. Let us, for example, consider a case where $p = 0.3p_0$. A static load which originates buckling amounts (in dimensionless magnitudes) to $\zeta' = 0.33$. If the load is impulsive and if $c\pi = 1.0$ (during the period of natural small oscillations of the shell the load intensity decreases by a factor of e) then, as it is shown in Figure 6, the dynamic load causing buckling is $\zeta' = 0.73$. Thus, the critical dynamic load is 2.2 times greater than the dynamic one. For loads whose intensity further decreases with time, this difference will be greater. For example, for $c\pi = 3.0$, the dynamic buckling load amounts to $\zeta' = 1.28$ which exceeds by 3.8 times the corresponding static load.

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